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Supporting Information for

**Connecting the SST Pattern Problem and the Hot Model Problem**

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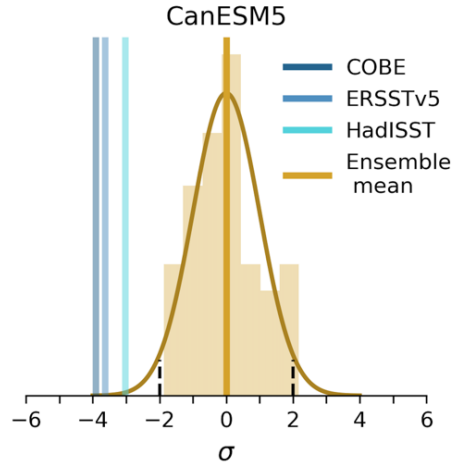
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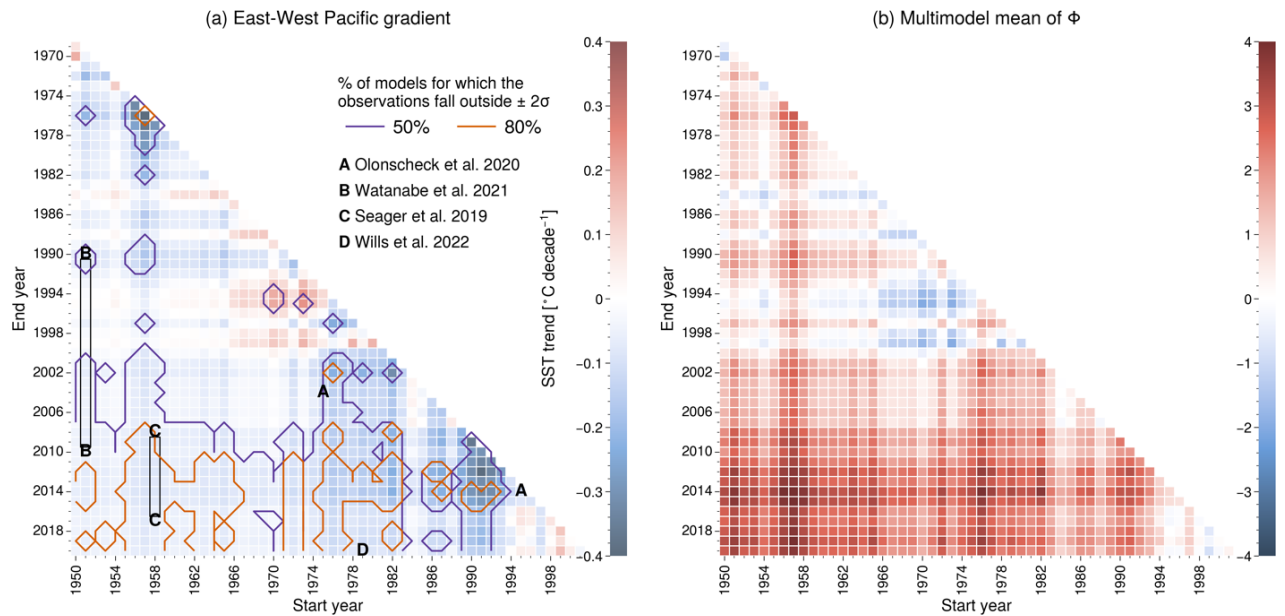
**Contents of this file**

Figures S1 to S5

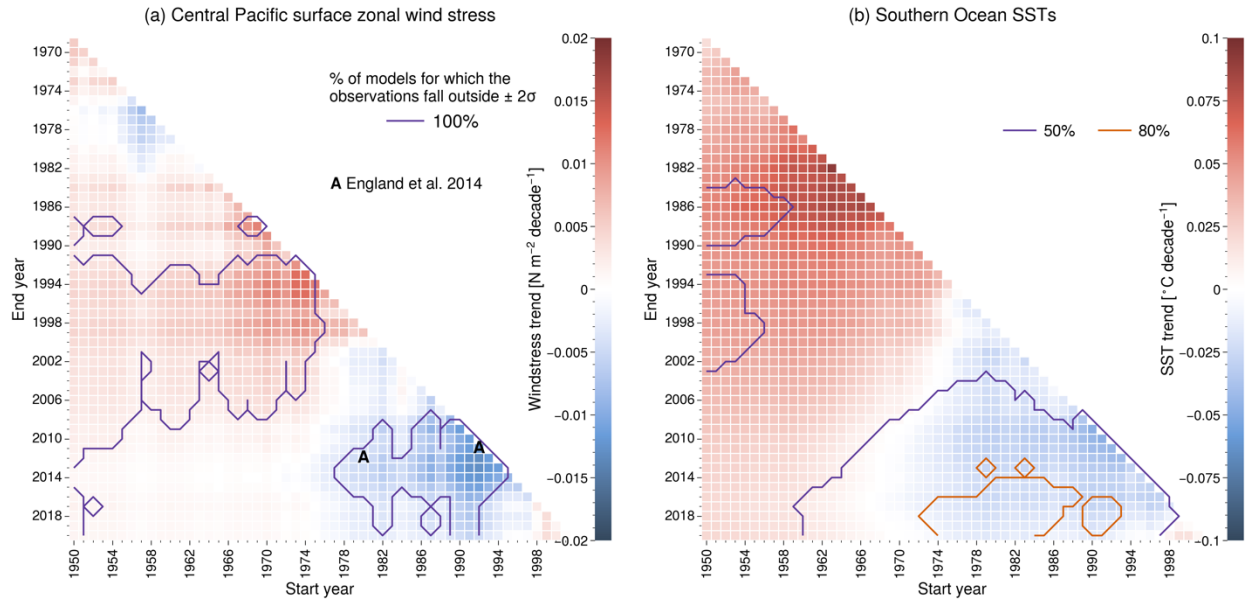
Tables S1



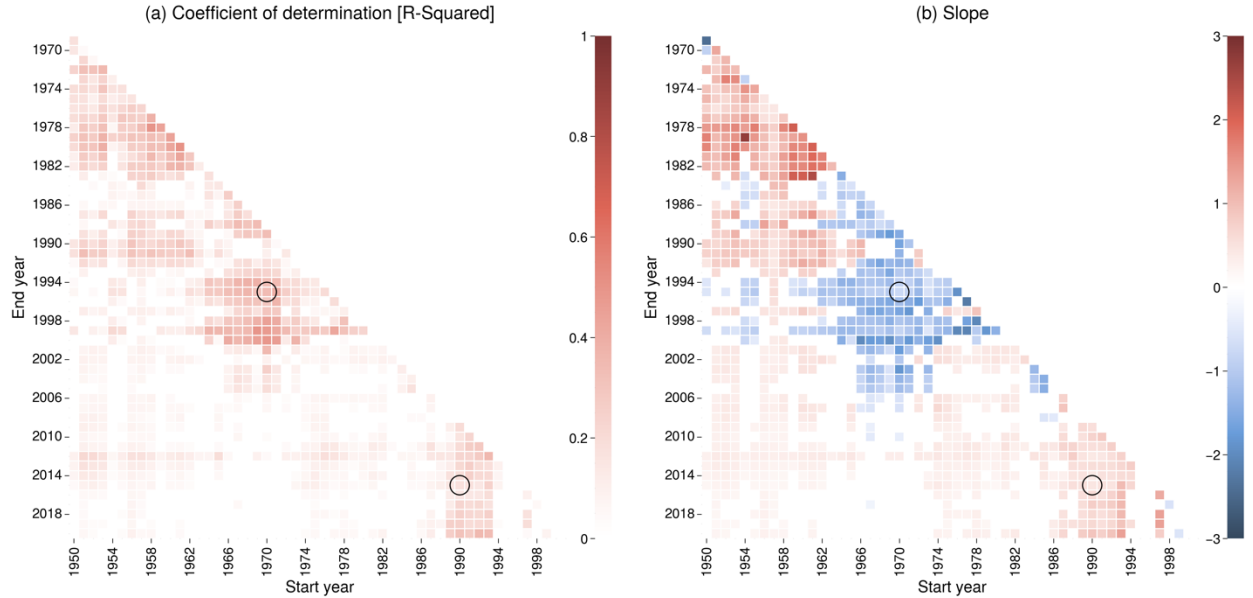
**Supplemental Figure 1** Illustration of  $\phi$ : Similar to main text Fig.2d but with all three observational products (which are averaged for figures in the main text). Fig. 1c shows the number of models for which the observations fall outside  $\pm 2$  standard deviations (dashed vertical lines) of the fitted normal probability density function of the ensemble simulations. The value of  $\phi$  that becomes problematic is subjective because the finite size ensembles do not perfectly sample the full model distribution, but our results hold for different choices of critical  $\phi$ .



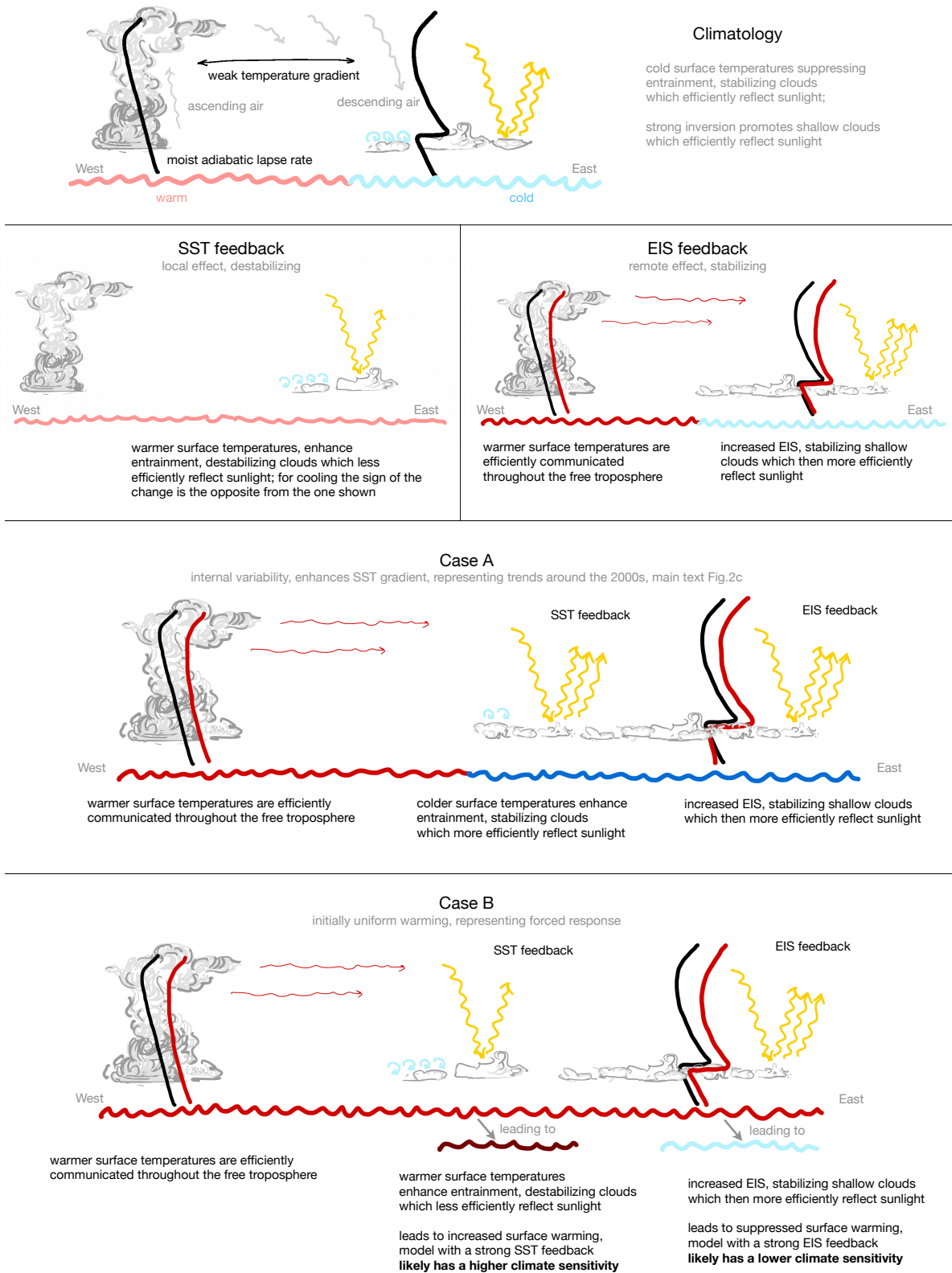
**Supplemental Figure 2** (a) As main text Fig.1b but overlaid with selected contours of main text Fig.1c. (b) Multi-model mean of  $\phi$ . For main text Fig.1c and 2b and c,  $\phi$  is computed for each single model.



**Supplemental Figure 3** (a) As in SM Fig.2a but for the Nino3.4 box zonal wind stress; (b) the Southern Ocean temperature trends. The Southern Ocean is defined as all ocean area poleward of 50° S.



**Supplemental Figure 4** (a) Coefficient of determination ( $R^2$ ) and (b) slope between EffCS and the location of the observations in the ensembles,  $\Phi$ , for all time scales and trends in the East-West Equatorial Pacific gradient. Note the weak correlation between the models' inability to capture the long-term trends and EffCS (lower left). Slopes indifferntiable from zero at the 95% confidence level are masked white in both panels.



**Supplemental Figure 5** Illustrative sketch of SST and inversion feedbacks and the representative cases A and B in the main text. The inversion feedback is called “EIS” here which refers to Estimated Inversion Strength.

**Table 1** Names of 15 CMIP large ensembles used in the main text. “historical” simulation end in 2005 (for CMIP5 models) and 2014 (for CMIP6 models). We use the indicated scenarios (RCP for CMIP5 models, SSP for CMIP6 models) for the years up to 2020. *N* indicates the number of ensemble members; *EffCS* indicates the effective climate sensitivity in *K*.

Model	Scenarios	N	EffCS (K)	Modeling center	Reference
ACCESS-ESM1.5	Historical, SSP5-8.5	40	3.88	Commonwealth Scientific and Industrial Research Organisation (CSIRO)-Bureau of Meteorology (BOM), Australia	Ziehn et al. 2020
CanESM2	Historical, RCP8.5	50	3.7	Canadian Centre for Climate Modelling and Analysis (CCCma), Canada	Kirchmeier-Young et al. 2017
CanESM5	Historical, SSP5-8.5	50	5.64	Canadian Centre for Climate Modelling and Analysis (CCCma), Canada	Swart et al. 2019
CESM1	Historical, RCP8.5	40	4.1	NSF-DOE-NCAR, United States of America	Kay et al. 2015 EffCS values from Meehl et al. 2013 – note that this is not the CMIP5 version
CESM2	Historical, SSP3-7.0	106	5.15	NSF-DOE-NCAR, United States of America	Rodgers et al. 2021
CNRM-CM6.1	Historical, SSP2-4.5	10	4.9	Centre National de Recherches Météorologiques (CNRM)- Centre Européen de Recherche et de Formation Avancée en Calcul Scientifique (CERFACS), France	Voltaire et al. 2019
CSIRO-Mk3.6	Historical, RCP8.5	30	4.09	CSIRO, Australia	Jeffrey et al. 2013
GFDL-CM3	Historical, RCP8.5	20	3.95	NOAA/GFDL, United States of America	Sun et al. 2018
GFDL-ESM2M	Historical, RCP8.5	30	2.44	NOAA/GFDL, United States of America	Rodgers et al. 2015
GISS-E2.1-G	Historical, SSP2-4.5	18	2.71	NASA-GISS, United States of America	Kelley et al. 2020
IPSL-CM6A-LR	Historical, SSP2-4.5	11	4.7	IPSL, France	Boucher et al. 2020
MIROC6	Historical, SSP5-8.5	50	2.6	Atmosphere and Ocean Research Institute (AORI)- National Institute for Environmental Studies (NIES)- Japan Agency for Marine-Earth Science and Technology (JAMSTEC), Japan	Tatebe et al. 2019
MIROC-ES2L	Historical, SSP2-4.5	30	2.66	AORI-NIES-JAMSTEC, Japan	Hajima et al. 2020
MPI-ESM	Historical, RCP8.5	100	2.8	MPI-M, Germany	Maher et al. 2019 note that this is not the CMIP5 version
NorCPM1	Historical, SSP2-4.5	30	3.03	Norwegian Climate Centre (NCC), Norway	Bethke et al. 2021